for a disk. The spread of the points around the functions (12) is 13% for a cylinder, 19% for a sphere, and 20% for a disk. The function (12a) yields values of Nu that are 13% higher than those form (11a). For a sphere, on the contrary, the function (12b) yields 20% lower Nu than does (11b). The experimental data obtained for a sphere and a cylinder can be considered to be in good agreement with the predictions from (11).

NOTATION

u, velocity; $|u_1|$, $|u_2|$, amplitudes of velocity components; ρ_0, gas density, r, x, radial and axial coordinates of the cylindrical coordinate system; t, time; ω, cyclic oscillation frequency; Sh = ωd/|u|; Re_{osc} = Ud/ν; d, characteristic size of the body (diameter of the sphere, cylinder, and disk); Pr, Prandtl number; f = ω/2π; R, tube radius; R_0, half-width of the jet; $\bar{x} = x/R$; u_{00}, $|u_{10}|$, $|u_{20}|$, amplitudes at $x = x_0$; ℓ, cylinder length (disk thickness); c_0, speed of sound; $\bar{a}$, constant component; $\bar{b}$, $\bar{c}$, first and second harmonics; u_{0m}, $|u_{1m}|$, $|u_{2m}|$, amplitudes of velocity components at the jet axis; $\bar{u}_1 = u_1/u_{1m}$, $\bar{u}_1 = |u_1|/|u_{1m}|$, $u_2 = |u_2|/|u_{2m}|$; U(x), range of velocity oscillations at the jet axis.

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HYDRODYNAMIC CHARACTERISTICS OF ASCENDING GAS—LIQUID FLOW

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A new equation for a cell model of bubble flow is obtained on the basis of a previously unused boundary condition. A comprehensive investigation of the hydrodynamic characteristics of this flow has been carried out on a specially built apparatus, enabling us to test experimentally both the newly derived equation and that published earlier by Marrucci. A comparison of the calculated and experimental data showed that the Marrucci equation describes the bubble flow more accurately.

The extensive use of gas—liquid flows in power, chemical, and biological engineering supports the constant interest in research into the various characteristics of the individual phases and of the flow as a whole. The monographs [1-3] have generalized the results of such research. Nevertheless, the accuracy in calculating (in a theoretical approach) such characteristics as the gas content and the absolute and relative velocities of bubble ascent in mass bubbling still remains inadequate.

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A cell model, which assumes the arbitrary partitioning of the entire volume of the gas–liquid system into a series of identical cells, each of which contains one bubble, has been used in a number of papers [4-7] to analyze the characteristics of gas–liquid flows. The interaction of this bubble with the others determines the choice of the specific boundary conditions at the surface of the cell. This means that the disturbance introduced into the stream by each bubble must be concentrated within the cell. Here the relation between the radii of the bubble and the spherical cell is assumed to be $a^3/b^3 = \alpha$.

It must be noted that the very formulation of the problem, which involves cells that do not actually exist, enables one to define the boundary conditions arbitrarily to some extent. In [4], for example, boundary conditions were used that Happle [8] proposed earlier for solid particles, in accordance with which the normal velocity component of the liquid vanishes at the outer boundary of a cell. Gal-Or [5, 6] has considered two versions: the shear stress τ and the curl of the velocity, curl V, vanish at the surface of a cell. These boundary conditions have also been encountered earlier in the description of solid particles [8]. Leclair and Hamilec [7] have analyzed the cell models proposed by various authors. It turned out that all the versions lead to calculating equations that predict a monotonic decrease in the velocity of bubble ascent with increasing gas content.

The results of Slobodov and Chepura [9] makes possible a sounder choice of boundary conditions. From the ideas of the cell model, they have written equations for the transport through the outer surface of a cell of fluxes of mass, momentum, angular momentum, kinetic energy, and the curl of the velocity, and they required that all these expressions vanish. Using the conditions at the surface of a particle, they have obtained a set of possible boundary conditions at the surface of a cell:

$$\tau = 0, \tag{1}$$

$$V_\theta = 0, \tag{2}$$

$$\mathrm{rot}\, V = 0. \tag{3}$$

As seen from these functions, a new, previously unstudied boundary condition (2) has appeared.

The indeterminacy in the boundary conditions is explained by the authors by the fact that simultaneous satisfaction of the conditions, in accordance with which all perturbations are closed within the cell, can be accomplished only if the cells can fill the entire volume of the three-dimensional space. And spheres, as is well known, produce some porosity in any packing.

The purpose of this paper is to analyze the possibility of using cell models to calculate gas–liquid flows, even when the boundary conditions (2) are used.

Assuming potential flow of liquid inside a cell, we solve the equation obtained by Lamb [10] for potential motion in flow over a sphere, together with the boundary conditions taken for the surface of the cell (Eq. (2)) and the bubble:

$$\frac{\partial \alpha}{\partial r} = W_{g\ell} \cos \Theta, \quad r = 0.$$

As a result of transformations, we find expressions for the radial and tangential components of the liquid velocity. Using them, together with the equation for the energy dissipated per unit volume of liquid [11], we obtain the final expression

$$\frac{W_{g\ell}}{W_\infty} = \frac{(2+\alpha)^2}{4(1-\alpha^{5/3})}, \tag{4}$$

reflecting the influence of the constraint of the bubble stream on the velocity of group ascent of the bubbles.

For an experimental test of different versions of the cell model, we built an experimental installation that produces ascending gas–liquid flow in an apparatus of the air-lift type with a rectangular cross section. The zone of ascending stream had dimensions of 95 × 25 × 400 mm. Air was supplied to the lower part of the apparatus, where a distributing device was mounted, consisting of a plate, covering the entire cross section of the ascending stream, in which 20 medical needles with an inside diameter of 0.8 mm were mounted to supply air, while there were also 85 openings 2 mm in diameter to supply water. The ends of the needles were elevated 15 mm above the distribution plate. Such a gas distributor construction made it possible to introduce the liquid and gas in a strictly vertical direction, which promoted an initial uniform distribution of the gas–liquid stream.

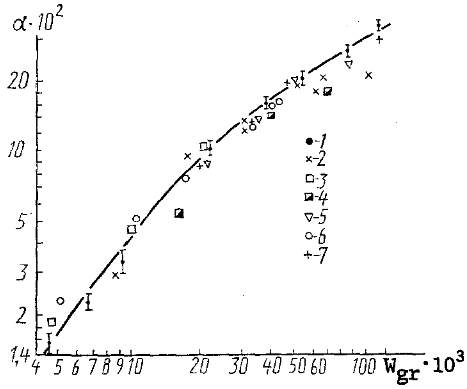

Fig. 1

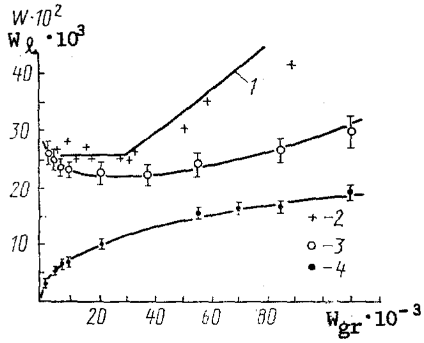

Fig. 2

Fig. 1. Gas content as a function of the reduced velocity of the gas: 1) our experimental data; 2) experimental data of [16]; 3) [17]; 4) [15]; 5) [18]; 6) [19]; 7) [20]. $W_{gr} \cdot 10^3$, m/sec; $\alpha \cdot 10^2$, m^3/m^3.

Fig. 2. Liquid velocity (W_ℓ) and bubble ascent velocity (W) as functions of the reduced gas velocity: 1 and 2) experimental data for W from [16] and [21]; 3) our experimental data for W; 4) our experimental data for W_ℓ. $W \cdot 10^2$, m/sec; $W_\ell \cdot 10^2$, m/sec.

The method of needle sensors, which is the best-founded and has been described in [12, 13], was used to study the gas content. From these papers it follows that the ratio of the volume of the gaseous phase to the volume of the gas–liquid mixture is equivalent to the ratio of the time spent by the gaseous phase at the point of the system under consideration (t_i) to the total measurement time (T):

$$\alpha = \frac{\Sigma \Delta t_i}{T}.$$

To measure Δt_i and T, a certain voltage is established between the needle sensor, fastened to the end of a moveable rod, and an electrode of arbitrary shape, placed so as not to distort the gas–liquid stream. At the instant the sensor enters a gas bubble, a pulse with a very steep front is produced in the measurement circuit. The gas content will then be determined by the total duration of these pulses relative to the observing time. An F 5041 frequency meter was used as the secondary instrument. The duration of one measurement was 10 sec. Measurements were made 10-15 times at each point and the results were averaged. The needle sensor was made of PETNKh-155 enameled Nichrome wire 0.12 mm in diameter. Measurements of α were made in four different cross sections, separated from the upper edge of the gas distributor by 50, 100, 200, and 300 mm. The investigated quantity was measured at seven points in each cross section.

In the statistical analysis of the measured results, the data obtained in one cross section were treated as a common set, consisting of several groups of data obtained at one point. In accordance with [14], we calculated the group, intragroup, and intergroup variances. On their basis, we determined the total corrected variance in the investigated cross section. Our analysis of the measured results showed that the gas content increases insignificantly with greater distance form the gas distributor. The influence of the height of the layer on the gas content has been studied in [15], where it was shown that in concurrent water–air flow, α almost always increases with greater distance from the gas distributor. Both in [15] and in our tests, this relation was not very strong, but stable.

In the analysis of the gas content of the entire layer as a whole, the data obtained for each cross seciton were treated as group data and were analyzed in the sequence indicated above. In detetermining the confidence interval here and below, the reliability was assumed to be 0.9. The results of the analysis are presented in Fig. 1. Literature data are also presented there. As seen from the figure, practically all the data fit fairly uniformly along the curve that approximates our results. We can thus state that our data are in satisfactory agreement with the literature data.

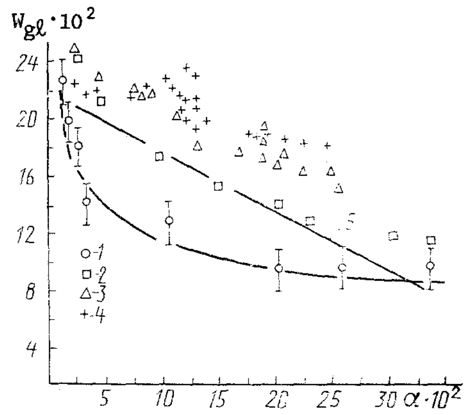

Fig. 3. Relative velocity as a function of gas content: 1) our experimental data; 2) experimental data of [32]; 3) [33]; 4) [25]; 5) [34].

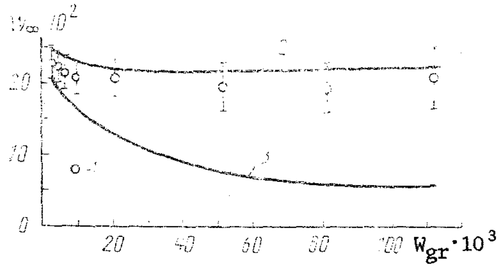

Fig. 4. Ascent velocity of a single bubble as a function of reduced gas velocity: 1) our experimental data; 2) calculation from Eq. (5); 3) calculation from Eq. (4). $W_\infty \cdot 10^2$, m/sec.

We used a somewhat modified method of needle sensors [13] to measure the velocity of the gas bubbles. For this we mounted two needles in the direction of the stream so that their ends were a certain distance $\Delta\ell$ apart. The velocity was measured as follows. A frequency meter, operating in the regime of measuring the time interval between pulses, is connected to each of the needles so that the signal for contact with a bubble from the first needle starts the time measurement while the signal from the second needle stops it, which enables us to determine the time T in which the bubble traveled the distance $\Delta\ell$ between the needles. We measured $\Delta\ell$ with a KM-6 cathetometer to within hundredths of a millimeter. The bubble velocity was measured at the same points where the gas content was investigated. Measurements were taken 12-20 times at each point.

A statistical analysis of the experimental results, made in the same sequence as in the investigation of gas content, showed that the bubble ascent velocity does not depend on the height of the layer. The results of the investigation of W at a distance of 200 mm from the gas distributor are given in Fig. 2. Our dependence of W on W_{gr} agrees well, on the qualitative level, with literature results. It was noted in [22], for example, that in a noncirculating column, in the absence of large-scale circulation (the reduced gas velocity was less than $40 \cdot 10^{-3}$ m/sec), the bubble ascent velocity decreases somewhat with increasing W_{gr}. But if stable, large-scale circulation loops are formed in the column with a descending liquid stream, then the bubble ascent velocity increases noticeably [23]. Similar data were noted in [24]. Our results at velocities up to $30 \cdot 10^{-3}$ m/sec (the bubbles can be assumed to be single under such conditions [25]) agree well with the generalized results of the studies cited in Wallis's book [26].

In addition to our data, the results of investigations in [16, 21] are presented in Fig. 2. As seen from the graphs, our data lie considerably lower those from these sources, since the construction of our experimental installation made it possible to produce small and practically identical bubbles.

All the foregoing permits us to hope that our data reflect the true picture of the gas-liquid layer.

A number of very complicated methods making it possible to measure the liquid velocity at any point of a bubble layer have been described in the literature [27-29]. Since in this stage of research we are interested in the mean value of the liquid velocity, we use a simpler method based on features of our test column. From the equality of the liqud flow rates in the ascending and descending streams, we can write:

$$W_{\ell} = \frac{2S_d W_{\ell d}}{S_a(1-\alpha)},$$

where S_d is the cross-sectional area of the one descending stream; S_a is the cross-sectional area of the ascending stream; $W_{\ell d}$ is the liquid velocity in the descending stream.

The unknown quantity $W_{\ell d}$ was determined from the velocity of migration of dye marker introduced into the descending stream. The velocity of migration of the marker as a function of the liquid circulation velocity was determined over a segment with a length of 50 or 100 mm, 10 times for each value of W_{gr}.

The existence of the previously obtained dependence of α on W_{gr} enabled us to represent the results of a calculation based on the equation in the form of the dependence of W_{ℓ} on W_{gr} (Fig. 2).

As seen from the figure, the liquid velocity constantly increases with an increase in the reduced velocity of the gas, but this relation gradually weakens, which is explained by the corresponding increase in hydraulic resistance along the liquid channel. The experimental results were compared with the few literature data. An apparatus very similar to ours was investigated in [18], for example, but the airlift was "complicated" by forced liquid circulation. Moreover, gas was introduced into the central part of the column in such a way as to create far less hydraulic resistance of the circulating liquid. These differences may explain the fact that the data obtained in [18] are considerably higher than ours (they are not plotted in Fig. 2). Data that fit far closer to our measurements are given in [30], but because of construction features of the experimental installation, they also lie above our experimental data.

The relative bubble velocity was calculated form the equation $W_{g\ell} = W - W_{\ell}$, using the data obtained earlier.

In calculating the confidence interval for $W_{g\ell}$, we allowed for the fact that W and W_{ℓ} were measured at different times and with different instruments. This allows us to consider these measurements to be statistically independent, and hence the confidence interval for the error in the indirect measurement of $W_{g\ell}$ can be calculated from the equation [31]:

$$\Delta W_{g\ell} = (\Delta W^2 - \Delta W_{\ell}^2)^{0,5},$$

where ΔW and ΔW_{ℓ} are the respective confidence intervals.

In Fig. 3 we give $W_{g\ell}$ as a function of α, since most literature data are presented in just this form. In the figure we also plot experimental data obtained on a circulating apparatus [32] and the results of experiments on special installations that produce one-dimensional bubble flow. In [33], for example, the experiments were carried out in a conical tube with a descending liquid stream. Here the conditions were chosen so that the gas bubbles stood still. In the second case [25], $W_{g\ell}$ was measured, after the gas supply was instantaneously turned off, from the velocity of ascent of the lower boundary of the bubble layer. Data obtained in noncirculating and airlift apparatus have not been found in the literature. Besides the experimental data, in Fig. 3 we present calculations from the equaiton [34]:

$$W_{g\ell} = W_{\infty}(1-\alpha)^2,$$

which is a generalization of the experimental data of several authors. As seen from Fig. 3, our data are located considerably lower, which is quite logical and is explained by the concurrent motion of liquid and gas. Moreover, in our case the gas is the motive power for the liquid.

We estimate the accuracy of theoretical equations for the cell model on the example of the Marrucci model [4], which has become the best known,

$$W_\infty = W_{g\ell}(1-\alpha^{5/3})/(1-\alpha)^2 \quad (5)$$

and Eq. (4), which we derived from an analysis of the new boundary conditions:

$$W_\infty = 4W_{g\ell}(1-\alpha^{5/3})/(2+\alpha)^2.$$

For this, substituting the experimental values of the gas content and the relative velocity into these equations, we calculate the ascent velocity of a single bubble for different values of the reduced velocity of the gas. We compare the calculated results with experimental literature data.

An important aspect of such an investigation is the estimate of the confidence interval ΔW_∞ for the ascent velocity of a single bubble (W_∞). Let us consider in more detail the process of determining ΔW_∞ on the example of an estimate of the accuracy of the Marrucci equation.

It is well known that the mean-square deviation of a function of several variables may be calculated from the equation [31]

$$\sigma_y^2 = (\partial f/\partial x_1)\,\sigma_{x_1}^2 + (\partial f/\partial x_2)\,\sigma_{x_2}^2 + \ldots + (\partial f/\partial x_i)\,\sigma_{x_i}^2, \quad (6)$$

where x_i is the i-th variable and σ_{x_i} is the mean-square deviation of the i-th variable.

The result of differentiating Eq. (5) in accordance with Eq. (6) has the form

$$\sigma_{W_\infty}^2 = \left(\frac{1-\alpha^{5/3}}{(1-\alpha)^2}\right)^2 \sigma_{W_{g\ell}}^2 = \left(\frac{2W_{g\ell}(1-\alpha^{5/3}) - \frac{5}{3}\,\alpha^{2/3}(1-\alpha)}{(1-\alpha)^3}\right)^2 \sigma_\alpha^2. \quad (7)$$

The quantities σ_α and $\sigma_{W_{g\ell}}$ appear in (7). We determined the first of these earlier in analyzing the results of measuring the gas content. We find the quantity $\sigma_{W_{g\ell}}$ by using the fact that $W_{g\ell}$ is, in turn, a function of the variables W and W_ℓ, i.e., we can also use Eq. (6) to determine it. As a result of differentiation, we obtain

$$\sigma_{W_{g\ell}}^2 = \sigma_W^2 + \sigma_{W_\ell}^2.$$

Since the quantities σ_W and σ_{W_ℓ} were determined earlier, there is no difficulty in calculating $\sigma_{W_{g\ell}}$. Substituting the values of $\sigma_{W_{g\ell}}$, σ_α, and α into Eq. (7), we calculate σ_{W_∞} and hence ΔW_∞ for all the values of W_{gr} used in the experiments. In Fig. 4 we give the ascent velocity of a single bubble as a function of the reduced gas velocity, and the corresponding confidence intervals. As seen from the figure, a definite tendency for the bubble ascent velocity to decrease may be traced at low reduced gas velocities, with subsequent stabilization and even some increase with increasing W_{gr}. A relation of this nature may be explained by the fact that in our experimental setup, the bubble diameter increases from 2.5 to 6.5 mm with increasing W_{gr}. This, in turn, results in a decrease in the ascent velocity (in a given range of bubble sizes) [26]. Using the dependence of the bubble diameter on the reduced gas velocity obtained in our experiment, and data from Wallis's book [26] on the ascent velocity of a bubble as a function of its diameter, we constructed the dependence of W_∞ on W_{gr} presented in Fig. 4. Comparing the experimental and calculated values of W_∞, one may see that the latter lie somewhat lower, but mainly within the confidence interval.

The results of calculations from Eq. (4), made in a similar sequence, are also given in Fig. 4. From the graph it is entirely obvious that the boundary conditions (2), and hence Eq. (4), lead to considerably understated results in comparison with experimental data.

Our research has thus shown that the use of the new boundary conditions in the cell model of bubble flow, and hence of the new equation, does not enable one to improve the accuracy in calculations of hydrodynamic characteristics. The equation proposed earlier by Marrucci turned out to be more accurate.

NOTATION

a, bubble radius; b, radius of a spherical cell; α, gas content; τ, shear stress at the surface of a cell; V_θ, tangential component of the liquid velocity at the surface of a cell;

$W_{g\ell}$, relative velocity of ascent of a bubble; W_∞, ascent velocity of a single bubble; W_{gr}, reduced velocity of the gas, calculated over the entire cross section of the ascending stream; W, bubble ascent velocity in mass bubbling; W_ℓ, liquid velocity in the ascending stream; θ, angle; r, current radius of a cell; mean-square deviation of a quantity.

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